**Risk and Return – Chapt. 10 in RWJJ**

## Converting Total Return to Annualized Return

Returns are measured over a period of time – usually over the course of a year, but if some other period, they can be annualized.

This is very similar to calculating the EAR, which we did in chapter 4

Example:

Buy a stock on Jan. 1 for $100

Sell the stock on April 1 for $102

What is the annualized return?

Simple Return = 102-100 = 102/100 – 1 = 2%

100

Note that since this was over a three-month period, this assumes quarterly compounding.

Annualized return assumes we had the same return for an entire year.

Annualized return with quarterly compounding = (1 + .02)4 – 1 = 8.24%

But we don’t have to assume quarterly compounding. We can assume continuous compounding.

Quarterly Return =  = 1.98% if we assume continuous compounding

Annualized Return with continuous compounding = (1.98%) (4) = 7.92%

Notice that in each case, the PV of 100 becomes a FV of 108.24 at the end of the year.

Another Example:

Suppose you buy a non-dividend-paying stock for $100, hold it for 5 years and then find that it is worth $155. Over that time period you have a simple return of 55%. Your annualized return has been:

(1 + .55)1/5 – 1 = 9.16% if we assume annual compounding

 = 43.8%/5 = 8.77% if we assume continuous compounding

5

Calculating Average Annual Returns and Standard Deviations from HISTORIC DATA

Average annual Return =  = Arithmetic mean =



Geometric Average Return = Geometric Mean = Buy and Hold Return

= [Π (1+Rt)]1/T – 1

Geometric Average Return – The compounded annual return earned by an investor who bought the security and held it for ‘T’ years. This is equivalent to earning this return each year and reinvesting the earnings at the end of each year.

**Example**: Let’s take the annual returns for a hypothetical stock which you bought at the beginning of 2001:

2001 5%

2002 9%

2003 -12%

2004 20%

Arithmetic Mean

‾= .05 + .09 - .12 + .20 = .22 = .055 = 5.5%

1. 4

Geometric Mean = [(1.05) (1.09) (0.88) (1.20)]1/4 – 1 = 4.85%

The Arithmetic Mean is approximately equal to the Geometric Mean plus one-half the variance. This means that the larger the variance of the data, the greater will be the difference between the arithmetic and geometric means.

If we want to calculate the Variance and Standard Deviation, we must use the arithmetic mean.

Variance of a population (using HISTORICAL DATA)

σ2 = variance = Average squared difference between actual return and average return. Always uses arithmetic average return



σ2 = (.05 - .055)2 + (.09 - .055)2 + (-.12 - .055)2 + (.20 - .055)2

4

= .000025 + .001225 + .030625 + .021025

4

= .0529 = .013225

4

σ = standard deviation = Positive square root of variance



= .115 = 11.5%

**Note though** that if these 4 years are only a sample of the annual returns for this stock and we wish to estimate the variance and standard deviation of the population (we can’t know them for sure without all the data), we must divide by T-1 instead of T. In this example, that would give us a sample variance of .0176 and a sample standard deviation of 13.279%.

Your text lists the average annual returns and standard deviations for the years

1926 – 2020. For small-cap stocks, large-cap stocks, and Treasury Bills they are:

**1926 – 2020**

### Investment Arith. Mean Return Standard Deviation

Stocks (small cap) 16.2% 31.3%

Stocks (large cap) 12.2 19.7

Treasury Bills 3.3 3.1

Note: ↑σ ⇒ ↑‾

First Definition of Risk = Variability of Returns = Total Risk

Basic Principle of Finance (one of them): There is an expected reward for bearing risk.

We use U.S. Treasury yields as our proxy for the risk-free rate.

Any return greater than a treasury’s return is a reward for bearing risk – We call it

a risk-premium.

The risk premium you come up with is determined, in part, by how you measure it.

Do we use 1926-2020, or some other time period?

Arithmetic mean or geometric mean returns?

Treasury Bond or Treasury Bill yields as the risk-free rate?

The key is – what do you **expect** the risk premium to be in the **future**?

The generally accepted risk premium for the S&P 500 is usually between 2% and 8%. For uniformity, we will use 5.7% in most of our examples.

What does Standard Deviation mean?

If we assume a normal distribution for the population (note that these 88 years are a sampling from that population), then we can predict the probability of a single year’s return for each security type.

Note: When we look at this year’s probabilities, we don’t look at the historic treasury yield as the risk-free rate. We look at today’s treasury yield. Let’s say that it’s approximately 3%.

So, using this method, the most likely return for stocks this year is:

3.0% Current Risk-free Rate

+ 5.7% Historic Risk Premium

8.7% Expected Return for Stocks - Middle of the Prob. Distrib.

And since about 2/3 of the probability distribution lies within one standard deviation of the expected value, we can say that there is a 2/3 chance that the S&P 500 will post a return between -11% and 28.4% this coming year.

Suppose you wanted to know what is the probability that the S&P 500 will return more than 25% next year.

If we assume that returns are normally distributed (they aren’t, but it is close enough for this sort of an exercise), we want to find the area of the probability distribution to the right of 25% when the mean is 8.7% and the standard deviation is 19.7%. As you remember from statistics, this involves calculating z-values and perhaps using tables to find the area under the curve. Fortunately, Excel does all this for us. If we go to NORMDIST in the Excel functions, it asks us for the value of X – which is the point we are looking for (in this case, it is .25). Then, we enter the mean of the distribution (.087) and the standard deviation (.197). Finally, under Cumulative, enter True.

The result will be the area under the normal curve to the left of the point in question (in our example, the probability you will get a return that is less than 25% is .796 – or about 79.6%). To find the area to the right, just subtract that value from 1.0. So there is about a 20.4% chance that the S&P will earn a return greater than 25% next year.