**Fall 2019**

Duration and Bootstrapping Assignment

1. On the Assignments page of our class website, you will find a link to the Bootstrapping website. Follow the “Instructions for Students” which will walk you through how to download and use the Bootstrapping Spreadsheet. When you have completed those instructions, answer the questions below:
	1. What is the price of a bootstrapped 2.2% (coupon rate) 10-year Treasury Note?

**102.392**

* 1. What is the YTM of the Treasury Note in question a?

**1.936%**

* 1. Price a 10-year 2.2% corporate bond so that it has a 50- basis point credit spread over the Treasury note you bootstrapped.

**97.92089**

* 1. Using the zero-coupon rates (the semiannual z values) that were found in the bootstrapping spreadsheet, find what the YTM would be for a previously-issued Treasury bond that matures in exactly two years and is currently selling at par value.

**1.88243%**

* 1. Repeat a-d above for a Treasury Note and a corporate bond with a 3.0% coupon rate.

**114.224**

**1.929%**

**109.4596**

**Same as before – 1.88243%**

* 1. Look at the Zero Coupon Yield Curve that the bootstrapping spreadsheet made. How would you describe the shape of this yield curve?

**Flat or slightly upward sloping**

* 1. Based on the Unbiased Expectations Theory of the term structure, what does this yield curve tell us about investors’ expectations over the next ten years?

**Short-term rates are expected to stay about where they currently are or short-term rates are expected to increase somewhat**

**Note that your answers for a-e may be somewhat different than mine depending on when you downloaded the spreadsheet. That’s ok. They should not be significantly different though.**

1. You manage a pension plan with $100 million in assets that have a duration of 12 years and $200 million in liabilities with a duration of 24 years.
	1. Calculate your duration gap

**DURGap = 12 – (200/100 x 24) = -36.00**

* 1. Are you hurt if interest rates go up, or if they go down?

**If interest rates go down, the value of both your assets and your liabilities will go up, but the value of your liabilities will go up by more, so you are hurt if interest rates go down.**

* 1. Suppose you want to execute a macro hedge using T-bond futures contracts traded on the CBOT. Do you want to go short or long?

**The futures contract will go up in value if interest rates go down. This will offset what happens to your pension plan. So go long.**

* 1. Suppose that the cheapest to deliver bond has a duration of 25.3 years. How many contracts will you short/long if you want to hedge your position as completely as possible?

**VF x DURF = -(VA x DURGap)**

**VF x 25.3 = -($100 million x -36.00) → VF = $142,292,490.12**

**At $100,000 per futures contract, go long 1,423 contracts**

**Or go long (buy) 1,423 contracts**

1. A friend of yours who manages the pension plan for a small New Orleans tech company has hired you as a consultant to help him select the right investments to meet the pension’s future obligations. Today, November 1, 2019, he acquired a liability of $20 million to pay on Nov. 1, 2024. Five-year bonds with the credit quality and liquidity that your friend is looking for are currently yielding 7.0% on a BEY basis. This means that the present value of the future liability is $14,178,376.27, assuming interest can be earned at the rate of 3.5% per semiannual period for the next five years. Conveniently, the yield curve is flat and will only move in parallel shifts. Your friend wants to fund the obligation with an investment that will assure him of having accumulated $20,000,000 by Nov. 1, 2024. It is crucial that he have at least $20 million by that date – no matter what interest rates do between now and then. That is the sole objective. He is prepared to invest exactly $14,178,376.27 and is considering three different bonds:

Bond #1: a 5-year, 7% coupon bond priced at par

Bond #2: a 6-year, 7% coupon bond priced at par that he would sell early

Bond #3: a 7-year, 8.25% coupon bond priced to yield 7% that he would sell early (note that you need to calculate the face value of this bond!)

Whichever bond your friend buys, he will spend exactly $14,178,376.27 today (that is the price of each of these three bonds). Note that the face value of each bond is not constrained be a ‘round’ number – in fact, with bonds 1 and 2, the face value must equal the price since those bonds are priced at par. For Bond #3, you will need to calculate the face value of the bond. Which bond do you recommend that your friend purchase?

1. On a spreadsheet, calculate the duration of each bond

**Bond 1 4.30384**

**Bond 2 5.00078**

**Bond 3 5.51210**

1. For each bond (on the spreadsheet), determine exactly how much money the pension fund will have after five years if interest rates (bond yields) go up to 8% immediately after he purchases the bond, or if they go down to 6% immediately after he purchases the bond. Assume that interest rates (bond yields) do not change after this initial change. If you do this correctly, your spreadsheet should be able to accommodate rate changes to values other than 8% and 6% as well. Be certain to remember that interest payments will be reinvested at current rates when received. That means that if rates drop to 6% immediately after the purchase, all coupon payments are reinvested at 6%.

 **6% 7% 8%**

**Bond 1 $19,867,248 $20,000,000 $20,136,325**

**Bond 2 $20,002,898 $20,000,000 $20,002,616**

**Bond 3 $20,103,866 $20,000,000 $19,905,932**

1. Present a recommendation of which of the three bonds that your friend should buy.

**Buy Bond 2**

1. Explain how the concept of duration allows you to pick the correct bond without doing any of these calculations.

**The duration of bond 2 matches the duration of the liability (5 years)**

1. The correct bond will provide the pension fund with more than $20 million if interest rates change. What property of the bond causes this?

**Convexity. The bond is more convex than the liability.**